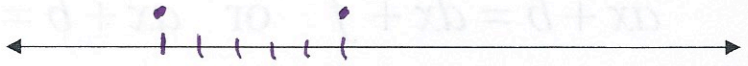
The absolute value of a number is its distance from zero on the number line, and distance cannot be negative.

What does $|x| = 5$ mean on the number line: 

Algebraic Definition of Absolute Value equations:

Let k represent a ***positive real number***

If $|x| = k$, then $x = k$ or $x = -k$. $x = \pm k$

If $|x| = 0$, then $x = 0$.

If $|x| = -k$, then no solution.

Rules to solve absolute value equations:

$|ax + b| = c$

$ax + b = c$ or $ax + b = -c$

The $|ax + b|$ must be the only thing on one side of the =, so get rid of any numbers not inside the bars. Add/subtract first and then multiply/divide. Watch for special cases.

Examples for Solving Absolute Value Equation:

Ex1: $|4x - 3| = 8$

$4x - 3 = 8$ $\quad +3 \quad +3$ $\quad 4x = 11$ $\quad x = \frac{11}{4}$

$4x - 3 = -8$ $\quad +3 \quad +3$ $\quad 4x = -5$ $\quad x = -\frac{5}{4}$

Ex2: $|4 - 2x| = 0$

$4 - 2x = 0$ $\quad -4 \quad -4$ $\quad -2x = -4$ $\quad x = 2$

Ex3: $3|3x + 7| - 5 = 10$

$+5 \quad +5$ $\quad \frac{3|3x+7|}{3} = \frac{15}{3}$ $\quad |3x + 7| = 5$ $\quad 3x + 7 = \pm 5$

$3x + 7 = 5$ $\quad -7 \quad -7$ $\quad 3x = -2$ $\quad x = -\frac{2}{3}$

$3x + 7 = -5$ $\quad -7 \quad -7$ $\quad 3x = -12$ $\quad x = -4$

Ex4: $2|1 + 5x| + 11 = 3$

$-11 \quad -11$ $\quad \frac{2|1+5x|}{2} = \frac{-8}{2}$ $\quad |1 + 5x| = -4$

No solution

Extraneous Solutions: A solution of an equation derived from the original equation but that is not really a solution of the original equation. This will occur when there is a variable on the right hand side of the equation as well as the left hand side of the equation.

Ex5: $|2x + 5| = 3x + 4$

$2x + 5 = \pm(3x + 4)$

$2x + 5 = 3x + 4$ $\quad -2x \quad -2x$ $\quad 5 = 1x + 4$ $\quad -4 \quad -4$ $\quad 1 = 1x$ $\quad x = 1$ ✓

$2x + 5 = -3x - 4$ $\quad +3x \quad +3x$ $\quad 5x + 5 = -4$ $\quad -5 \quad -5$ $\quad 5x = -9$ $\quad x = -\frac{9}{5}$

Check to see if answer is a solution by plugging into the orig. eqtn.