

Chapter 1.5 Solving Absolute Value Inequalities

Solving an absolute value inequality is very similar to solving an absolute value equation.

Start by isolating the absolute value on one side of the inequality symbol, then follow the rules below:

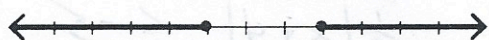
If the symbol is $>$ (or \geq): Then use the word (or). Called a disjunction

If $a > 0$, then the solutions to $|ax + b| > c$ are $ax + b > c$ or $ax + b < -c$.

If $c < 0$, all real numbers will satisfy $|ax + b| > c$.

Think about it: absolute value is always positive (or zero), so, of course, it is greater than any negative number.

Graph:



Solve and graph the absolute value inequality:

Ex 1: $|2x - 7| - 3 > 6$

$$\frac{|2x - 7|}{+3} > \frac{9}{+3}$$

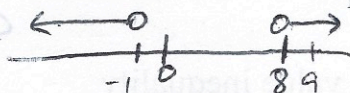
$$\frac{2x - 7}{+7} > \frac{9}{+7}$$

$$x > 8$$

$$\frac{2x - 7}{+7} < \frac{-9}{+7}$$

$$2x < -2$$

$$x < -1$$



Ex 2: $3|2x - 5| + 6 \geq -9$

$$\frac{3|2x - 5|}{3} \geq \frac{-15}{3}$$

$$|2x - 5| \geq -5$$

all real #s

Assignment: Solve and graph the absolute value inequality

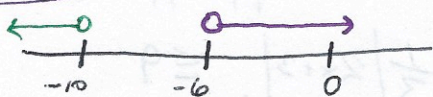
1. $|k + 8| > 2$

$$\frac{k + 8}{-8} > \frac{2}{-8}$$

$$k > -6$$

$$\frac{k + 8}{-8} < \frac{-2}{-8}$$

$$k < -10$$



2. $|3w - 8| \geq -2$

all real #s

Absolute value greater than a negative #

3. $|3c + 4| \geq 7$

$$\frac{3c + 4}{-4} \geq \frac{7}{-4}$$

$$3c \geq 3$$

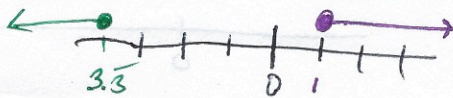
$$c \geq 1$$

$$\frac{3c + 4}{+4} \leq \frac{-7}{+4}$$

$$3c \leq -11$$

$$c \leq -\frac{11}{3}$$

$$c \leq -3\frac{2}{3}$$



4. $|5y - 2| + 5 > 2$

$$\frac{|5y - 2|}{-5} > \frac{-3}{-5}$$

all real #s

If the symbol is $<$ (or \leq): Then use the word (and). Called a conjunction

If $a > 0$, then the solutions to $|ax + b| < c$ are $ax + b < c$ and $ax + b > -c$.
Also written: $-c < ax + b < c$.



Graph:

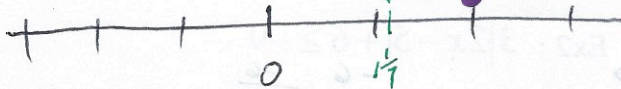
If $c < 0$, there is no solution to $|ax + b| < c$.

Think about it: absolute value is always positive (or zero), so, of course, it cannot be less than a negative number.

Ex3: $|7x - 11| \leq 3$

$$\begin{array}{r} 7x - 11 \leq 3 \\ +11 \quad +11 \\ \hline 7x \leq 14 \\ x \leq 2 \end{array}$$

$$\begin{array}{r} 7x - 11 \geq -3 \\ +11 \quad +11 \\ \hline 7x \geq 8 \\ x \geq \frac{8}{7} \end{array}$$



Ex4: $\frac{1}{2}|x + 4| + 7 < 2$

$$\begin{array}{r} \frac{1}{2}|x + 4| + 7 < 2 \\ -7 \quad -7 \\ \hline \frac{1}{2}|x + 4| < -5 \\ |x + 4| < -10 \end{array}$$

No solution

Assignment: Solve and graph the absolute value inequality

5. $|2x - 9| < -8$

No solution

Absolute value less than a negative #

6. $|3d - 5| \leq 10$

$$\begin{array}{r} 3d - 5 \leq 10 \\ 3d \leq 15 \\ d \leq 5 \end{array}$$

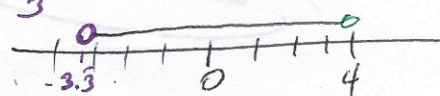
$$\begin{array}{r} 3d - 5 \geq -10 \\ 3d \geq -5 \\ d \geq -\frac{5}{3} \\ d \geq -1\frac{2}{3} \end{array}$$

7. $2|1 - 3h| - 10 < 12$

$$\begin{array}{r} 2|1 - 3h| - 10 < 12 \\ +10 \quad +10 \\ \hline 2|1 - 3h| < 22 \\ |1 - 3h| < 11 \end{array}$$

$$\begin{array}{r} 1 - 3h < 11 \\ -1 \quad -1 \\ \hline -3h < 10 \\ -3 \quad -3 \\ \hline h > -\frac{10}{3} \end{array}$$

$$\begin{array}{r} 1 - 3h > -11 \\ -3h > -12 \\ -3 \quad -3 \\ \hline h < 4 \end{array}$$



8. $\frac{1}{3}|2x + 3| - 1 \leq 8$

$$\begin{array}{r} \frac{1}{3}|2x + 3| - 1 \leq 8 \\ +1 \quad +1 \\ \hline \frac{1}{3}|2x + 3| \leq 9 \\ |2x + 3| \leq 27 \end{array}$$

$$\begin{array}{r} 2x + 3 \leq 27 \\ -3 \quad -3 \\ \hline 2x \leq 24 \\ x \leq 12 \end{array}$$

$$\begin{array}{r} 2x + 3 \geq -27 \\ -3 \quad -3 \\ \hline 2x \geq -30 \\ x \geq -15 \end{array}$$

